Temporal Scaling Laws for Wetting and Drying Processes in Variably Saturated Soils

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Background and motivation

Scaling laws facilitate the capture and description of inherent patterns within a system, revealing similarities and information transfer across various scales. They promote a better understanding of the fundamental principles that govern the dynamics of natural systems.

Scaling laws have been extensively discussed in the literature, with power laws associated with various phenomena. One of the fundamental scaling laws in hydrology is between drainage area and mean river discharge, known as Hack's law [Hack, 1957].

The application of scaling laws to the modeling of subsurface water flow remains rare. This study aims to develop a mathematical relationship, such as a power law, that incorporates key factors such as soil texture, hydraulic properties, and initial and boundary conditions such as infiltration rate and water table depth. This model would estimate the magnitude of quasi-steady state timescales for a wetting process at a constant infiltration rate and a drainage process achieving a hydrostatic pressure profile.

Methodology

Theory of infiltration into a variably saturated subsurface with a water table boundary condition

$$\phi \frac{\partial S_w(\psi_p)}{\partial t} = \nabla \cdot q$$

$$q = -k_s k_r \nabla(\psi_p - z)$$

$$K_r(\psi_p) = \frac{\left(1-(\alpha\psi_p)^{n-1}\left(1+(\alpha\psi_p)^n\right)^{-(1-1/n)}\right)^2}{\left(1+(\alpha\psi_p)^n\right)^{\frac{(1-1/n)}{2}}}$$

$$\theta(\psi_p) = \theta_r + \frac{(\theta_s-\theta_r)}{\left(1+(\alpha\psi_p)^n\right)^{(1-1/n)}}$$

Ψ_p: subsurface pressure head [L];
z: depth below the surface [L];
K_s: saturated hydraulic conductivity [LT^{-1}];
K_r: relative hydraulic conductivity [-];
S_w: degree of the saturation [-];
ϕ: porosity [-];

α: Van Genuchten parameter;
n: Van Genuchten parameter;
S_{sat}: relative saturated saturation;
S_{res}: relative residual saturation;
θ_s: saturated water content;
θ_r: residual water content

In this study, the numerical model PARFLOW was utilized to solve the 3D Richards equation, implementing the Van Genuchten soil texture model [Richards, 1931; Van Genuchten, 1980].

Experimental setup

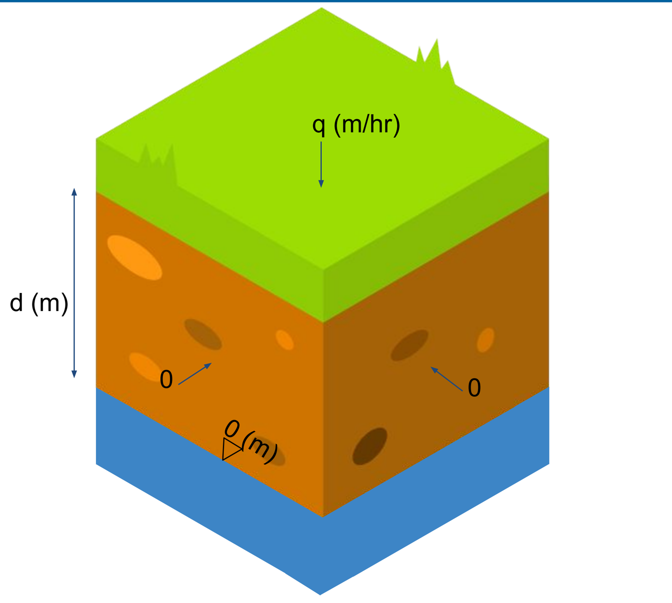

Fig. 1. A scheme of the study domain

The study focuses on a 1D vertical flow process. 1000 random combinations of the following soil properties, initial and boundary conditions were simulated:

- $10 \geq d \geq 1$ m
- $10^{-1} \geq q \geq 10^{-4}$ m/hr
- 12 soil types:
  $0.29 \geq K_s \geq 0.0002$,
  $0.46 \geq \Phi \geq 0.36$,
  $14.5 \geq \alpha \geq 0.5$

Experimental setup (cont.)

Boundary conditions	Infiltration phase	Drainage phase
Neumann-type boundary $-K_s K_r \nabla(\psi_p - z) = q_{bc}$	q_{top} = q m/hr (infiltration rate) $q_{bottom} = q_{lateral}$ = 0 m/hr	$q_{top} = q_{bottom} = q_{lateral}$ = 0 m/hr
Dirichlet-type	h_d = 0m	h_d = 0m
Initial condition	Hydrostatic profile	Infiltration pressure profile

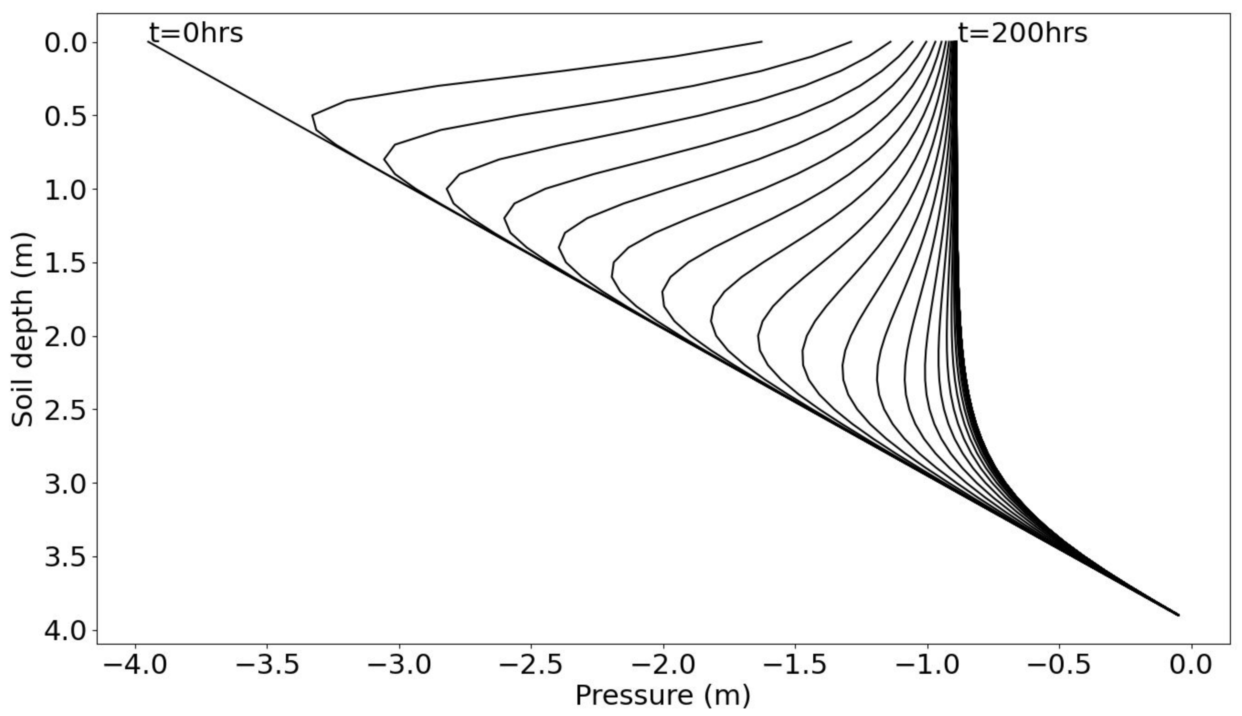

Fig. 2. An Example pressure profile for a 4m water table depth, 0.2m/hr saturated hydraulic conductivity, and 0.002m/hr infiltration rate.

The infiltration phase focuses on the time required for the soil to transition from a hydrostatic profile to a quasi-steady state under constant boundary conditions. While the drainage phase focuses on the interruption of infiltration, i.e. the transition from the quasi-steady state achieved in the infiltration phase back to a hydrostatic profile.

Scaling law for a 1D vertical infiltration

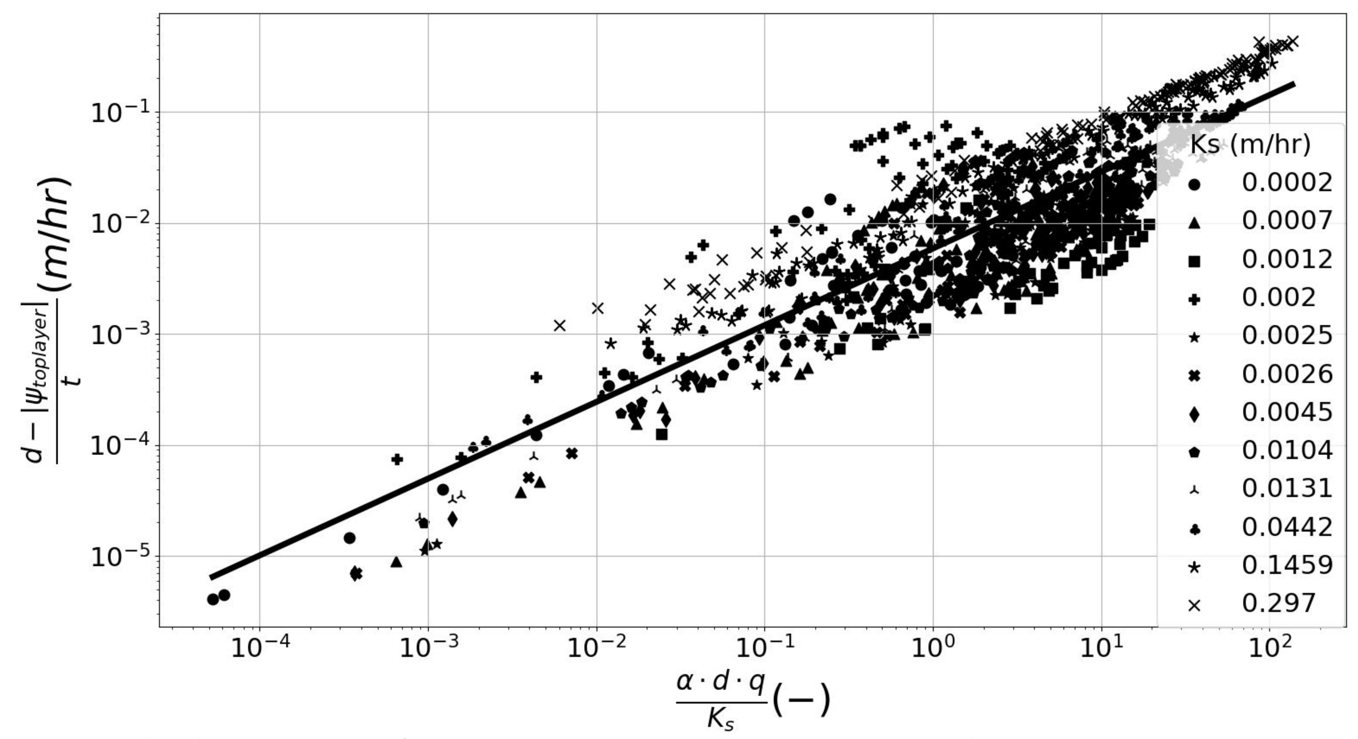

Fig. 3. Relationship of the considered parameters implementing top layer velocity at the steady state of the infiltration process.

$$\frac{d-|\psi_{top\,layer}|}{t} = 0.01\left(\alpha \cdot d \cdot \frac{q}{k_s}\right)^{0.69}$$

- Fitting accuracy $R^2 = 0.74$
- Variations from the fitting line can be attributed to the wide range of soil types.
- The solution is based on the velocity of the top layer pressure to reach a quasi-steady state.

Scaling law for drainage to groundwater

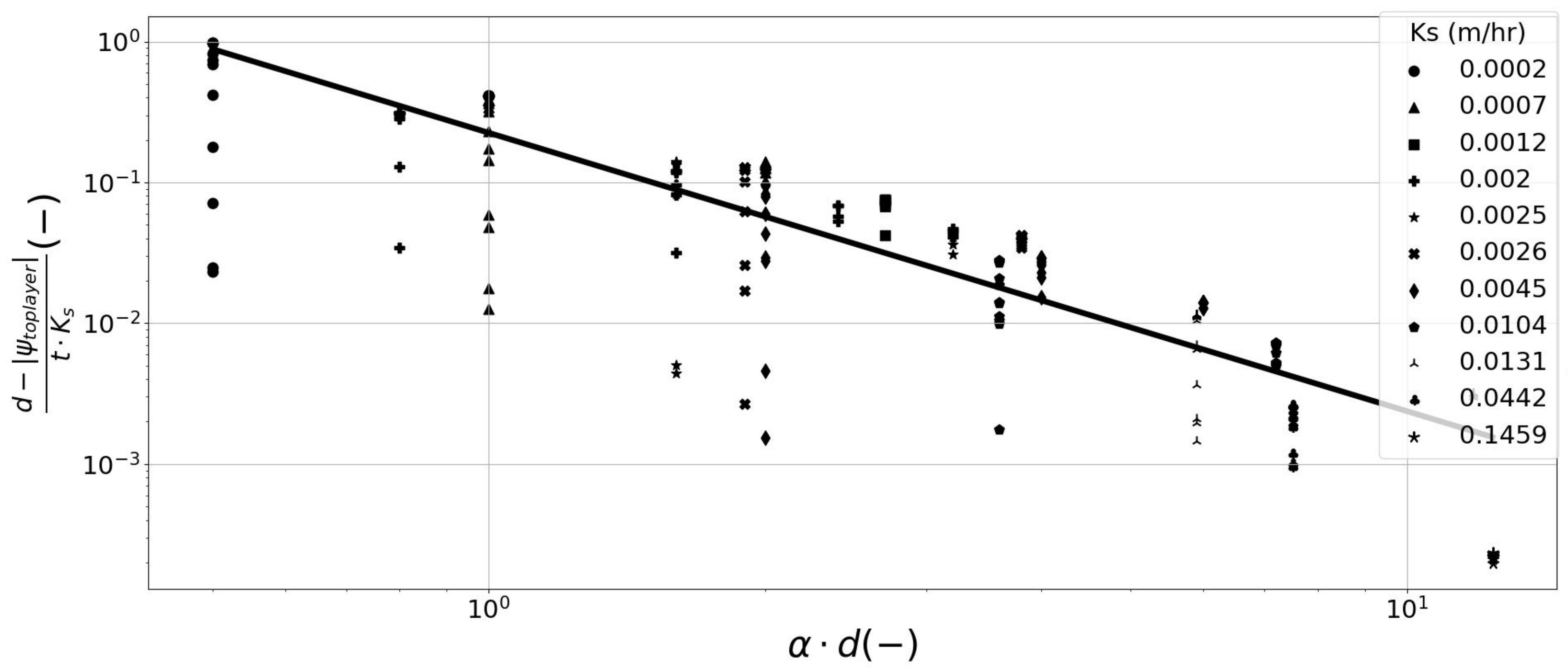

Fig. 4. Drainage process steady state scaling law implementing top layer velocity with a fitting accuracy R^2=0.74.

$$\frac{d-|\psi_{top\,layer}|}{t \cdot K_s} = 0.23(\alpha \cdot d)^{-1.98}$$

Scaling law for drainage to groundwater (cont.)

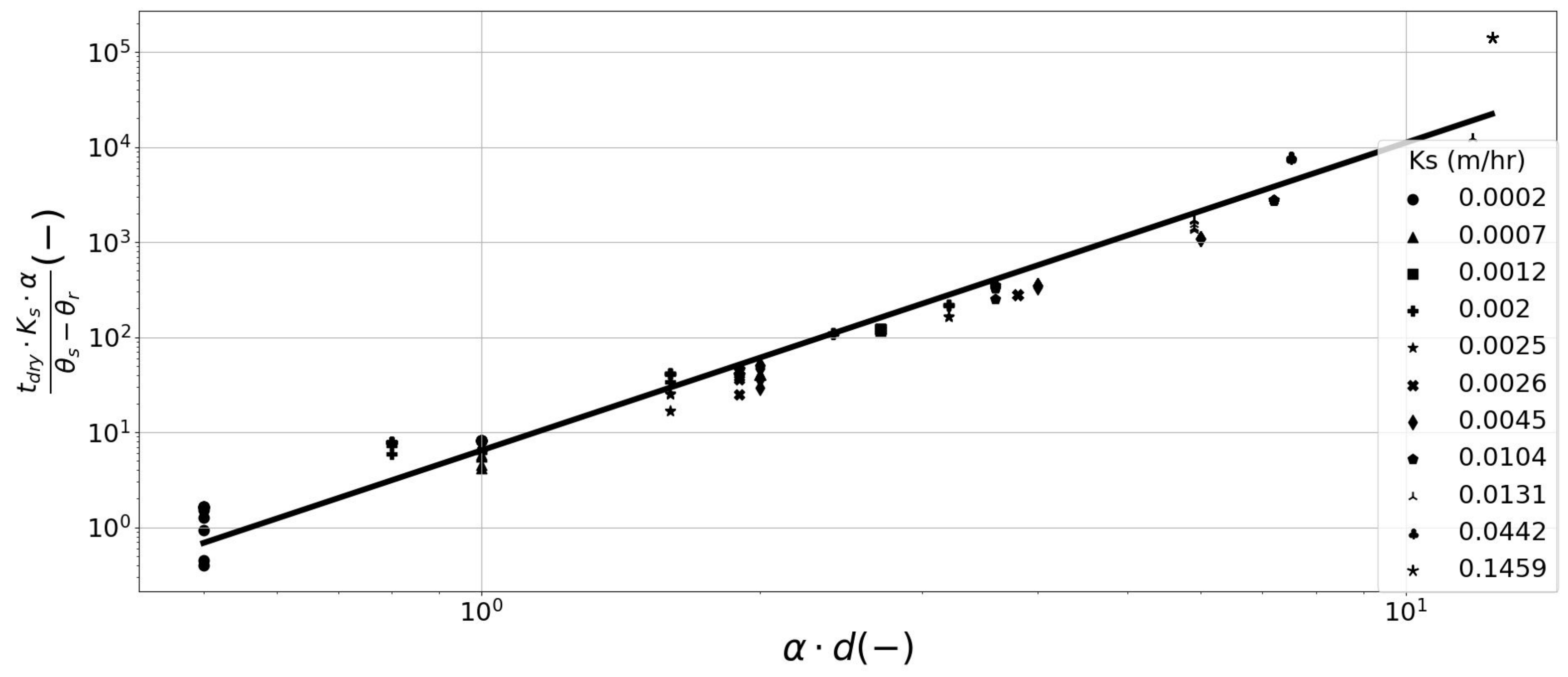

Fig. 5. Drainage phase scaling law based on the dimensionless time parameter by Srivastava & Yeh [1991]

$$\frac{t_{dry} \cdot K_s \cdot \alpha}{\theta_s - \theta_r} = 6.44(\alpha \cdot d)^{3.24}$$

- The resulting scaling law, with a coefficient of determination R^2 of 0.95, provides a dimensionless relationship among the key parameters examined in this study, inspired by the dimensionless time parameter by Srivastava and Yeh (1991).
- Both dimensionless correlations are applicable across various systems and scales, remaining invariant under changes in measurement units, facilitating extrapolation across different scales and orders of magnitude.

Conclusions

- The derived scaling law for the wetting process, based on the velocity of the top layer pressure, provides a comprehensive framework applicable to soil wetting across varying water table depths, diverse soil textures, and hydraulic conductivities.
- For the drainage phase, two dimensionless power-laws providing a broad understanding of timescales spanning multiple orders of magnitude are obtained.
- All derived relationships provide valuable information considering different initial and boundary conditions, allowing the investigation of the impact of infiltration rate, water table depth, soil texture and hydraulic properties on the wetting and drying timescales.
- Future research could extend the obtained results by considering initial conditions with varying moisture levels.

References

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